TE WAVE PROPERTIES OF SLAB DIELECTRIC GUIDE BOUNDED BY NONLINEAR NON-KERR-LIKE MEDIA

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ABSTRACT

TE wave properties of slab dielectric waveguide with nonlinear non-Kerr-like substrate and cladding are presented. The guide can support even- and odd-symmetrical modes in asymmetrical substrate and cladding and asymmetrical modes in completely symmetrical cases. The dispersion relations, electric field profiles are illustrated and analytically discussed in detail.

INTRODUCTION

It has been already known since the early 1980's [2]-[5] that the properties of waves guided by thin films take on striking new properties when one or more of the bounding media exhibits an intensity-dependent refractive index. With appropriate material conditions, both propagation wave-vector and the field distributions become strongly power-dependent. Because a number of potential applications for such nonlinear waveguides to all-optical signal processing have been identified, in recent years, with the development of technology, increasing attention has been devoted to these effects with a view to realizing these optical devices. These developments, in turn, have recently stimulated more realistic theoretical investigations of the properties of the nonlinear guided waves [1] [6]-[12]. With some exceptions (see e.g. [1][4][8][9]), many theoretical studies of nonlinear guided waves have been limited to Kerr-like nonlinear media (see e.g. [2][3][5][6][12] and references therein). Moreover, useful solutions of the nonlinear wave equation which includes a field-dependent dielectric constant have been obtained for the Kerr-like case.

However, in real media, many materials exhibit a refractive index which varies with the electric field raised to a power other than two [1] [4]. The actual dependence of the index on the optical field is intimately related to the physical process which gives rise to the nonlinearities, for example, for semiconductors, diffusion and recombination effects, etc. In retrospect, many numerical methods have been employed to analyze nonlinear slab-guided wave phenomena with non-Kerr-like media.

Although numerical methods are generally effective, they present also the disadvantage of not allowing physical interpretation of the solutions. Using purely numerical methods, a qualitative analysis of the structure is not possible and many of its underlying features can not be perceived.

In this paper the slab guide with the nonlinear cladding and substrate both having non-quadratic power-law dependent refractive index are analytically studied, and closed form solutions will be given for the first time. It indicates that there are even- and odd-symmetric modes in this structure not only for symmetric cladding and substrate, but also for asymmetric cases. The propagation properties of the guide are illustrated and discussed.

ANALYTICAL SOLUTIONS

The geometry considered in this paper is shown in Fig.1.

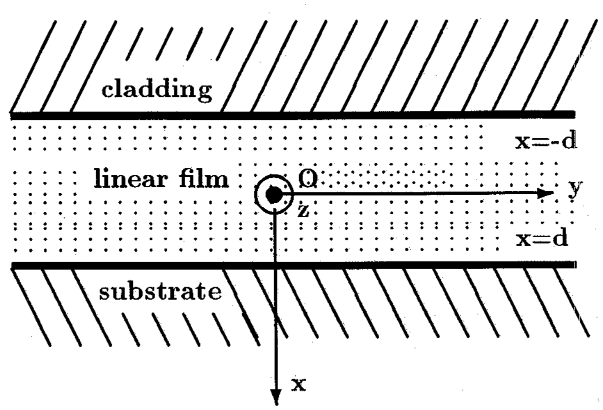

Fig.1: The nonlinear wave guide geometry used for the analysis.

It is assumed that the film region ($-d < x < d$) has

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a linear refractive index n_f (related relative dielectric constant is ϵ_f) that is independent of the wave power. Both of the cladding and substrating media can exhibit a field-dependent relative dielectric constant of the form $\epsilon_c = n_c^2 + \alpha_c|E_c|^{\delta_c}$ or $\epsilon_s = n_s^2 + \alpha_s|E_s|^{\delta_s}$ for the cladding and substrating medium, respectively. Fields are TE polarized having the form [4] $E(\vec{r},t) = Re\{E(x)e^{\imath(\omega t-\beta z)}\}$, only the component E_y of the electric field is non zero.

Using c, f, s referring to cladding, film and substrate, respectively, the field must satisfy the adjoint wave equations [1][4]. With $k_o^2 = \omega^2\mu_o\epsilon_o$, $\alpha_i (i = c, s)$ nonlinear coefficient, and $\alpha_i > 0 (i = c, s)$ for a focusing medium, and $\alpha_i < 0 (i = c, s)$ for a de-focusing medium, and using the normalized parameters recommended by Rozzi et al. [1]

$$k_c^2 = N^2 - n_c^2, \quad k_f^2 = n_f^2 - N^2, \quad k_s^2 = N^2 - n_s^2, \quad (1)$$

$$X = k_o x, \quad N = \frac{\beta}{k_o}, \quad a = k_o d, \quad (2)$$

and

$$\left.\begin{array}{l} u_c(X) = |\alpha_c|^{1/\delta_c} E_{yc}(X), \\ u_f(X) = E_{yf}(X), \\ u_s(X) = |\alpha_s|^{1/\delta_s} E_{ys}(X), \end{array}\right\} \quad (3)$$

the wave equations become:

$$\ddot{u}_c(X) - (k_c^2 - \sigma_c|u_c|^{\delta_c})u_c(X) = 0, \quad X \leq -a \quad (4)$$
$$\ddot{u}_f(X) + k_f^2 u_f(X) = 0, \quad -a \leq X \leq a \quad (5)$$
$$\ddot{u}_s(X) - (k_s^2 - \sigma_s|u_s|^{\delta_s})u_s(X) = 0, \quad a \leq X \quad (6)$$

where for focusing media $\sigma_i = 1$ and for de-focusing media $\sigma_i = -1$ $(i = c, s)$. $\ddot{()} = \frac{d^2()}{dX^2}$. The solution for focusing media is [8][9]:

$$u_c(X) = \pm \frac{(k_c^2 \frac{2+\delta_c}{2})^{1/\delta_c}}{\{\cosh^2[\frac{\delta_c}{2}k_c(X_c - a - X)]\}^{1/\delta_c}}. \quad (7)$$

Similarly, the solution of $u_s(X)$ in the substrate is:

$$u_s(X) = \pm \frac{(k_s^2 \frac{2+\delta_s}{2})^{1/\delta_s}}{\{\cosh^2[\frac{\delta_s}{2}k_s(X_s - a + X)]\}^{1/\delta_s}}. \quad (8)$$

For de-focusing media ($\sigma_i = -1$), analytical solutions can be also obtained using the function sinh() to replace the function cosh(). Here X_c and X_s are parameters related to the initial conditions. In (7) and (8) $X_i (i = c, s)$ can be positive or negative and zero. But for solutions of de-focusing media X_c and X_s must be positive, because the function $1/\sinh(w)$ has a pole at $w = 0$. If $\delta_i = 2$ $(i = c, s)$ (Kerr-law dependence), solutions (7) - (8) are the same as those given in [2][3]. Using these analytical solutions the waveguide shown in Fig.1 has been studied.

RESULTS

For symmetric modes (even modes) there is $E_f(-a) = E_f(a)$, and for anti-symmetric modes (odd modes) there is $E_f(-a) = -E_f(a)$. The solution of guided-waves in linear films is well-known:

$$\left.\begin{array}{ll} u_f(X) = A_f \cos(k_f X), & \text{even} - \text{modes} \\ u_f(X) = A_f \sin(k_f X), & \text{odd} - \text{modes} \end{array}\right\} N < n_f. \quad (9)$$

Using the boundary conditions at $X = \pm a$, for focusing medium condition

$$\left.\begin{array}{l} k_c \tanh(\frac{\delta_c X_c k_c}{2}) = k_s \tanh(\frac{\delta_s X_s k_s}{2}) \\ \frac{u_c(-a)}{\alpha_c^{1/\delta_c}} = \frac{u_s(a)}{\alpha_s^{1/\delta_s}} \frac{u_s(a)}{\alpha_s^{1/\delta_s}} \end{array}\right\} \quad (10)$$

must be satisfied.

If $n_c = n_f$, $\delta_c = \delta_s$, $\alpha_c = \alpha_s, X_c = X_s$, conditions (10) are automatically satisfied. Now we pay our attentions to the asymmetric case. For simplicity only the case $\delta_c = \delta_s = \delta$ is discussed.

That is, if $n_c \neq n_s$, we can choose a suitable parameter pair α_c, X_c and α_s, X_s to satisfy (10). Now the cladding and the substrate are not symmetric, but the structure can support even- and odd-symmetric modes. For a given cladding and substrate, only the modes which have the normalized propagation constant N determined by (10) can exist in this structure symmetrically or anti-symmetrically. As an example, in Fig.2 the dependencies $\alpha = \sqrt{\alpha_c/\alpha_s}$ vs X_s are drawn for $X_c = 10., n_c = 1.55, n_s = 1.54, n_f = 1.57, N = 1.560099, \sigma_c = \sigma_s = 1$ and $\delta = 0.5, 1.0, 1.5, 2.0, 2.5$ and 3.0. All of the values in the curves satisfy the condition (10) and can support both even- and odd-modes which have the same values of N and $a = k_o d$.

Therefore, for completely asymmetric structures even- and odd-modes can also propagate in nonlinear guides unlike in linear cases! For other cases $\sigma_c \neq \sigma_s$ and $\delta_c \neq \delta_s$ similar results can also be obtained.

If condition (10) is not satisfied, there will be no even- and odd-symmetric modes in the structure. The parameters $X_i (i = c, s)$ are dependent on the initial conditions and are not material nor structure parameters. If the material of the cladding and the substrate are really identical, that is, $\delta_c = \delta_s, n_c = n_s, \alpha_c = \alpha_s$, and $X_c \neq X_s$. That means only asymmetrical modes can now exist in the structure.

In linear cases if and only if the cladding and the substrate are identical, even- and odd-modes can propagate in the guide. For Kerr-like cladding and substrate ($\delta_c = \delta_s = 2$) our conclusion is consistent with the results given by Boardman et al. ([5] pp. 1703) who found that there are unusual asymmetric waves in a completely symmetric structure.

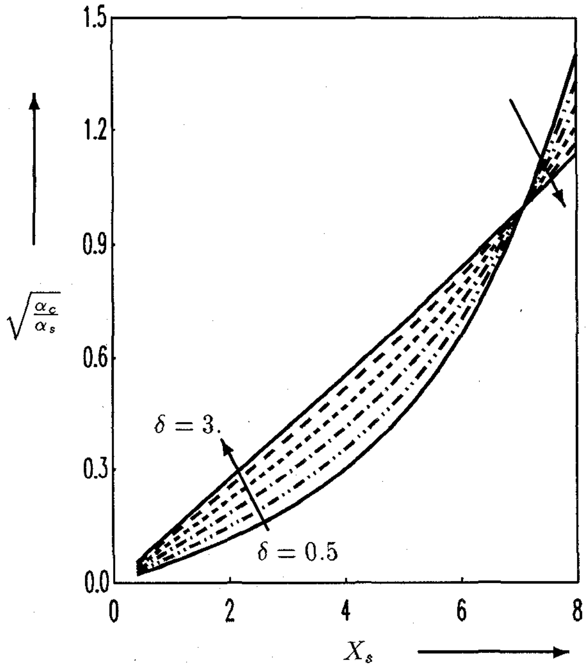

Fig.2: $\sqrt{\frac{\alpha_c}{\alpha_s}}$ vs X_s for $X_c = 10.$, $n_c = 1.55$, $n_s = 1.54$, $n_f = 1.57$, $N = 1.560099$, $\sigma_c = \sigma_s = 1$ and $\delta = 0.5, 1.0, 1.5, 2.0, 2.5$ and 3.0.

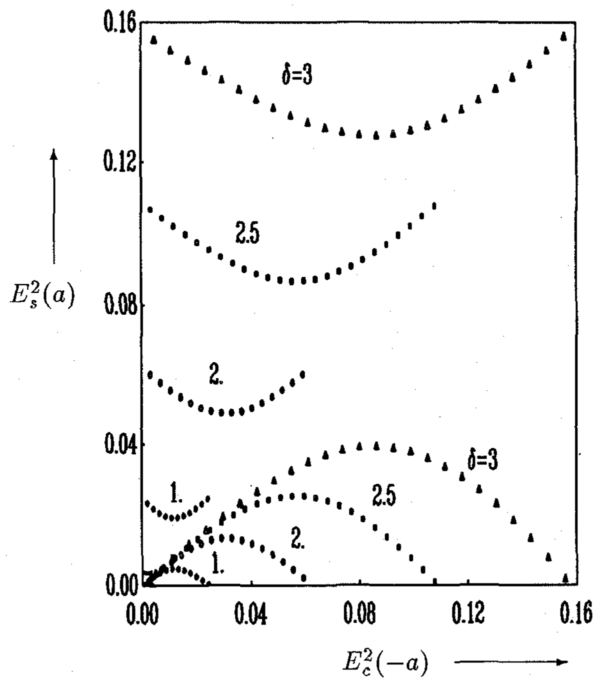

Fig.3: Phase portrait of the asymmetric structure: $n_f = 1.57, n_c = 1.55, n_s = 1.54, \frac{\alpha_s}{\alpha_c} = 1.5$.

In Fig.3 the phase portrait for an asymmetric structure is given. It indicates that for each set of determined structure parameters there are two possible branches. For the completely symmetric structure the phase portrait is shown in Fig.4. It shows that there are either symmetric modes (diagonal line) or unusual asymmetric modes (other lines). Here $\delta_c = \delta_s$. From Fig.4 it may be recognized that for a given value of $E_c^2(-a)$ there are two values of $E_s^2(a)$, one for the symmetric mode and the other for the asymmetric mode. That is, only the modes whose electric fields at the boundaries ($x = \pm d$), $E_s^2(a)$ and $E_c^2(-a)$ locate on the curves can exist in this structure unlike in linear cases where the modes having arbitrary values of the electric field at the boundaries $x = \pm d$ can propagate.

In the following discussion, it is always assumed that $n_c > n_s$. Now the cut-off condition is $k_c = 0$ or $N = n_c$. For linear cases dispersion curves of guided waves in the $N \sim a$ plane are a set of discrete curves, but for nonlinear cases, because there are many parameters for each given mode m, dispersion curves can vary in a region, named allowed region. In Fig.5 the allowed and forbidden regions are given for $n_c = 1.55, n_s = 1.54, n_f = 1.57$. In the region below the solid lines the guided waves have no maximum in the cladding or substrate. In the region above the solid lines the guided waves have a maximum in one region of the cladding and the substrate or in both regions. All dispersion curves with possible δ_c, δ_s, X_c and X_s are located in the allowed regions.

CONCLUSION

We have systematically examined the effect of guided waves in nonlinear waveguides. Nonlinear media which exhibit field-dependent refractive index in which the index change is proportional to the field raised to some arbitrary power were analytically studied. It is obviously from the analytic expressions that the design of nonlinear guided wave devices which rely on power-dependent changes in the field distribution depends strongly on the form of the nonlinearities.

The results show that for the completely symmetric structure there are unusual asymmetric modes and for an asymmetric structure there are possible symmetric modes. If a correct field distribution with related initial parameters is launched in the absence of loss, the wave should maintain that field distribution as it propagates down the waveguide. The explicit analytical expressions and the illustrations given here can be used for the design of optical devices based on nonlinear waveguide structures.

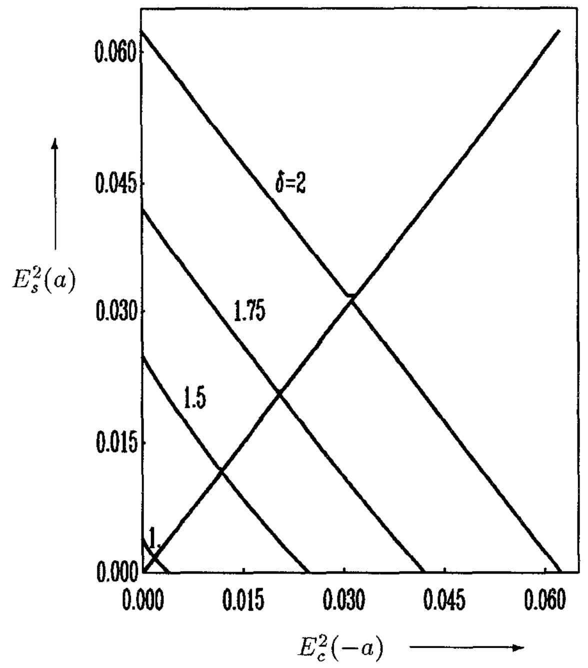

Fig.4: Phase portrait of the completely symmetric structure: $n_f = 1.57, n_c = n_s = 1.55$, $\frac{\alpha_s}{\alpha_c} = 1$.

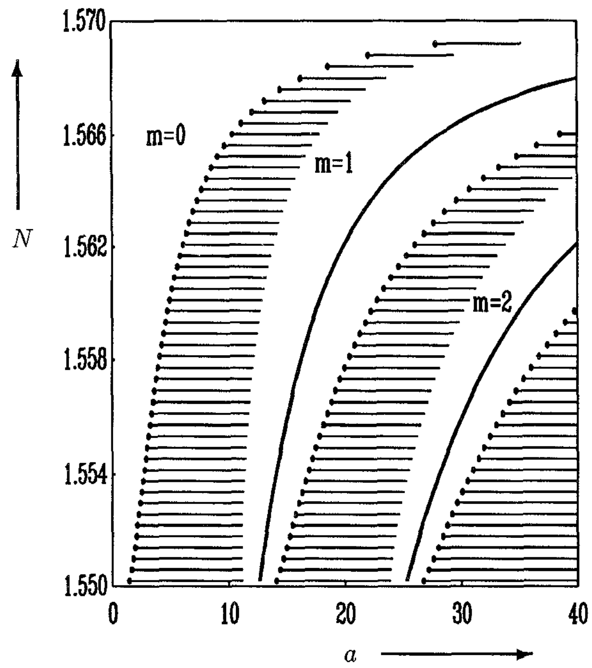

Fig.5: The allowed and forbidden bands. Shadded region: forbidden regions; —: $X_c = X_s = 0$; • • ••: linear curves.

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